Classical negation in a dynamic alternative semantics

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Links

Slides

https://patrl.keybase.pub/lenls2020.pdf

Draft paper "Towards a predictive logic of anaphora"

https://ling.auf.net/lingbuzz/005562

Introduction

- Dynamic Semantics (DS) as a logic of (singular) anaphora to indefinites (Heim 1982, Kamp 1981, Groenendijk & Stokhof 1991).
- Deficiencies of dynamic approaches:
  - Empirical wrinkles, with a particular focus on negation and disjunction.
  - Explanatory adequacy.
- A more principled logic of anaphora:

  Partial Dynamic Alternative Semantics (P-DAS).
- Reigning in P-DAS in the pragmatic component.

Discourse anaphora:

(1) A^1 philosopher attended this talk.
She$_1$ was sitting in the back.

Donkey anaphora:

(2) Everyone who invited a^1 philosopher
was relieved that she$_1$ came.

Defining characteristics of a dynamic logic

Egli's theorem:

$$(\exists^n \phi) \wedge \psi \Leftrightarrow \exists^n (\phi \wedge \psi)$$

Egli's corrolary:

$$(\exists^n \phi) \rightarrow \psi \Leftrightarrow \forall^n (\phi \rightarrow \psi)$$

(desirability of Egli's corrolary questionable — donkey sentences can have weak, existential readings; see Kanazawa 1994)

Many flavors of DS that fulfill these desiderata, and at least two separate traditions.

We'll focus on Dynamic Predicate Logic (DPL); it's logical properties are well understood, and it constitutes a foundation for much subsequent work in DS (see, e.g., Groenendijk, Stokhof & Veltman 1996 and van den Berg 1996).

Without going into too much detail of how it works — there'll be enough theory-building later — i'll discuss some empirical problems for DPL and related theories.

Dynamic semantics and its discontents

(3) #I haven't met any^1 philosopher.
She$_1$ was unwell.

(4) #No^1 philosopher attended this talk. She$_1$ was unwell.

Generalization

An indefinite in the scope of negation is *inaccessible* as an antecedent for a subsequent pronoun.

In DPL, the semantics of negation is tailored to derive this generalization.

Without going into the details of the DPL interpretation schema, negation kills any Discourse Referents (DRS) in its scope — it's a *destructive* operator.

The logic is such that, once dead, a DR can't be revived.

Double negation in DPL

This makes bad predictions (Groenendijk & Stokhof 1991, Krahmer & Muskens 1995, Gotham 2019, etc.).

(5) It's not true that NO^1 philosopher attended this talk. She$_1$'s sitting in the back!

DPL doesn't validate Double Negation Elimination (DNE); we want a logic of anaphora that validates DNE.

Destructive negation in DPL hamstrings the logic in other ways too; consider Partee's famous "bathroom" sentence.

(6) Either there is no^1 bathroom, or it$_1$'s upstairs.

(6) feels like it should be explicable via the logic of presupposition satisfaction (Beaver 2001), but due to the problem of DNE, this won't work in a DPL-like system.

In other words, we want to explain the anaphoric bathroom sentence in terms of the following:

(7) Either there is no bathroom, or
there isn't no^{1} bathroom and
it$_1$'s upstairs.

The treatment of negation in DPL — although motivated by accessibility generalizations — precludes this move.

In DPL, the semantics of the logical connectives is *tailored* to account for generalizations about where anaphora are licensed. e.g., it's built into the meaning of conjunction that the first conjunct is processed before the second.

(8) a. Someone^1 arrived already and she$_1$'s outside.

b. #She$_1$'s outside and someone^1 arrived already.

Unlike in the domain of presupposition projection, there are basically no competing approaches with the same of better empirical predictions.

The empirical problem of negation, and the conceptual issues may seem rather removed.

As we'll see however, solving the negation problem will involve adopting a simple, trivalent semantics for negation. This will give us a direction to pursue.

In the following, i'll outline a new, predictive dynamic logic, extending DPL. I'll dub this logic Partial Dynamic Alternative Semantics (P-DAS).

Partial Dynamic Alternative Semantics

Like Groenendijk & Stokhof, we'll give a dynamic interpretation for a simple predicate calculus, with natural numbers as variable symbols, and a priveleged tautology ε^n.

We'll treat sentential meanings as *mappings from assignments, to truth-value assignment pairs* (an enrichment of DPL meanings).

The truth-functional substract will be *trivalent*, so we'll pair output assignments with one of three truth-values, *true* ($\top$), *false* ($\bot$), and *maybe* ($\#$).

Partial assignments

We assume throughout that assignments are *partial* functions: $\mathbb{N} \mapsto D$.

In P-DAS, a pronoun indexed n (translated as variables) will induce a *presupposition* that n is defined at the input assignment.

We encode using Beaver's (2001) δ–operator.

δ	
1	1
0	#
#	#

Table 1: Beaver's (2001) δ-operator

A monadic predicate with a variable argument:

$$[\![P\ n]\!]^g := \{\, (\delta\ (n \in \text{dom } g) \wedge g_n \in I(P), g) \,\}$$

A monadic predicate with a constant argument:

$$[\![P\ c]\!]^g := \{\, I(c) \in I(P), g) \,\}$$

These clauses are generalized in an obvious way to n-ary predicates and sequences of terms.

It will frequently be useful to consider the interpretation of a sentence relative to a privileged *initial assignment* ($g_\top$).

This is the unique assignment whose domain is $\varnothing$; it reflects a state in which no variables have been introduced.

Relative to $g_\top$, a sentence will a free variable will always output the maybe-tagged input:

$$[\![P\ 1]\!]^{g_\top} = \{\,(\#, g_\top)\,\}$$

If the input is defined for g, the polarity of the output depends on whether or not g_1 is a P.

$$[\![P\ 1]\!]^{[1\mapsto a]} = \{\,(a \in I(P), [1 \mapsto a])\,\}$$

In order to model the contribution of indefinites, we introduce a privileged tautology: *random assignment* (ε^n) (van den Berg 1996: ch. 2).

$$[\![\varepsilon^n]\!]^g = \{ (\top, g^{[n \mapsto x]}) \mid x \in D \}$$

Assuming a simple domain of individuals $D := \{ a, b, c \}$, the effect of random assignment is illustrated below.

$$[\![\varepsilon^1]\!]^{g_\top} = \{ (\top, [1 \mapsto a]), (\top, [1 \mapsto b]), (\top, [1 \mapsto c]) \}$$

Presupposition satisfaction

Variables introduce indexed *presuppositions* that are satisfied by a preceding co-indexed *random assignment*.

Random assignment doesn't *just* satisfy the presupposition of subsequent variables, but also induces referential uncertainty relative to a set of alternatives (here: D).

In order to take the logic further, we next need to define negation and conjunction, but first some important background.

In a logic with three truth-values, ignoring the dynamic scaffolding, what is the semantic contribution of the logical operators?

The strong Kleene recipe: take the classical, bivalent operators, and their truth/falsity conditions, e.g.

- $\neg \phi$ is true if ϕ is false; $\neg \phi$ is false if ϕ is true.
- $\phi \wedge \psi$ is true if ϕ is true and ψ is true; $\phi \wedge \psi$ is false if either ϕ is false or ψ is false.

Where these conditions are silent, assume *maybe*; this is simply the logic we get if we interpret # as standing in for uncertainty between true and false.

The Strong Kleene truth-tables for ¬ and ∧

$\neg^s$	
1	0
0	1
#	#

$\wedge^s$	1	0	#
1	1	0	#
0	0	0	0
#	#	0	#

Table 2: Negation and conjunction in strong Kleene

Note: uncertainty projects whenever the truth/falsity conditions are silent; this means that if either conjunct is false, the whole conjunction is false, regardless of the truth value of the other conjunct.

Negation in P-DAS

Negation in P-DAS is just lifted strong Kleene negation:

$$[\![\neg\, \phi]\!]^g = \{\, (\neg^s\, t, h) \mid (t, h) \in [\![\phi]\!]^g \,\}$$

Dynamic Alternative Semantics (DAS) will swiftly become difficult to reason about.

It will be useful to define two auxiliary notions: the *positive* and *negative* extension of a sentence.

Definition (Positive and negative extension)

$$[\![\phi]\!]^g_+ = \{\, h \mid (\top, h) \in [\![\phi]\!]^g \,\}$$

$$[\![\phi]\!]^g_- = \{\, h \mid (\bot, h) \in [\![\phi]\!]^g \,\}$$

For completeness, we can also define the *maybe extension*:

$$[\![\phi]\!]^g_u = \{\, h \mid (\#, h) \in [\![\phi]\!]^g \,\}$$

Some helpful equivalences

We can think of P-DAS as consisting of two DPL-like logics, computed in tandem.

Based on the definition of negation, we already can see some useful equivalences:

$$[\![\neg\,\phi]\!]^g_+ = [\![\phi]\!]^g_-$$
$$[\![\neg\,\phi]\!]^g_- = [\![\phi]\!]^g_+$$
$$[\![\neg\,\phi]\!]^g_u = [\![\phi]\!]^g_u$$

N.b. on this basis that DNE is valid:

$$[\![\neg\,\neg\,\phi]\!]^g_+ = [\![\neg\,\phi]\!]^g_- = [\![\phi]\!]^g_+$$
$$[\![\neg\,\neg\,\phi]\!]^g_- = [\![\neg\,\phi]\!]^g_+ = [\![\phi]\!]^g_-$$

Towards conjunction

To understand conjunction in this logic, we'll start by defining lifted strong Kleene conjunction.

$$[\![\phi \vartriangle \psi]\!]^g = \{ (t \wedge^s u, i) \mid \exists h[(t, h) \in [\![\phi]\!]^g \wedge (u, i) \in [\![\psi]\!]^h] \}$$

It will be helpful to consider how to compute the postitive and negative extension of lifted strong Kleene conjunction.

+/− of lifted Strong Kleene conjunction

(9) a. $[\![\phi \triangle \psi]\!]^g_+ = \{\, i \mid \exists h[h \in [\![\phi]\!]^g_+ \wedge i \in [\![\psi]\!]^h_+]\,\}$

b. $[\![\phi \triangle \psi]\!]^g_- = \{\, i \mid \exists h[h \in [\![\phi]\!]^g_- \wedge (i, *) \in [\![\psi]\!]^h]\,\}$
$\cup \{\, i \mid \exists h[(h, *) \in [\![\phi]\!]^g \wedge i \in [\![\psi]\!]^h_-]\,\}$

How do we arrive at this?

- The verification conditions of $\wedge^s$ say that both conjuncts must be true, so we do relational composition of the positive extension of each conjunct.
- The falsification conditions of $\wedge^s$ just say that either conjunct must be false, so to cover all cases, we compose the negative extension of the first conjunct with all extensions of the latter, and vice versa.

Finalizing conjunction

Lifted $\wedge^s$ doesn't by itself give us a reasonable dynamic logic (ask me in the question period why).

To finalize the entry for conjunction, we need the *positive closure* operator †:

$$\begin{aligned}
[\![\dagger\,\phi]\!]^g = \; & \{\,(\top, h) \mid h \in [\![\phi]\!]^g_+\,\} \\
& \cup \{\,(\bot, g) \mid [\![\phi]\!]^g_+ = \varnothing \wedge [\![\phi]\!]^g_- \neq \varnothing\,\} \\
& \cup \{\,(\#, g) \mid [\![\phi]\!]^g_+ = [\![\phi]\!]^g_- = \varnothing \wedge [\![\phi]\!]^g_u \neq \varnothing\,\}
\end{aligned}$$

How to understand this: † ensures that DRS are only introduced in the positive extension.

Conjunction is defined via lifted strong Kleene + positive closure. The other binary connectives will be defined using the same method.

$$\phi \wedge \psi \Leftrightarrow \dagger\, (\phi \vartriangle \psi)$$

The positive extension is the same as lifted $\wedge^s$, but the negative extension is a test of the negative extension of $\wedge^s$:

(10) a. $[\![\phi \wedge \psi]\!]^g_+ = [\![\phi \vartriangle \psi]\!]^g_+$

b. $[\![\phi \wedge \psi]\!]^g_- = \{\, g \mid [\![\phi \vartriangle \psi]\!]^g = \varnothing \wedge [\![\phi \vartriangle \psi]\!]^g_- \neq \varnothing \,\}$

It's obvious that Egli's theorem is validated wrt the positive extension, since conjunction in the positive dimension is just DPL conjunction.

(11) a. $(\varepsilon^1 \wedge P\ 1) \wedge Q\ 1$

b. $\varepsilon^1 \wedge (P\ 1 \wedge Q\ 1)$

Egli's theorem is validated in the negative dimension too, thanks to the fact that conjunction is defined in terms of positive closure. I won't show this here (but see my paper).

Negation and accessibility

Recall, negation renders an indefinite inaccessible as an antecedent for future pronouns.

Despite the fact that negation in P-DAS is just lifted $\neg^S$ (and hence externally dynamic) we still capture this, due to positive closure.

(12) a. It's not true that anyone^1 is here.

b. $\neg\,(\varepsilon^1 \wedge H\ 1)$

Positive extension of the contained sentence:

$$[\![\varepsilon^1 \wedge H\ 1]\!]^g_+ = \{\, g^{[1\mapsto x]} \mid x \in I(H) \,\}$$

Now, to compute the negative extension. First, observe that the negative extension of random assignment is empty (it's a tautology).

$$[\![\varepsilon^n]\!]^g_+ = \{\, g^{[1\to x]} \mid x \in D \,\}$$
$$[\![\varepsilon^n]\!]^g_- = \varnothing$$

Based on this, we only need to concentrate on the case where the first conjunct is true. That means the negative-extension is only non-empty if the second conjunct is false.

(13) a. $[\![\varepsilon^1 \wedge H\ 1]\!]^g_-$

b. $= \{ g \mid [\![\varepsilon^1 \wedge H\ 1]\!]^g_+ = \varnothing \wedge [\![\varepsilon^1 \mathbin{\underline{\wedge}} H\ 1]\!]^g_- \neq \varnothing \}$

c. $= \{ g \mid I(H) = \varnothing \wedge \exists x [x \notin H\ 1] \}$

d. $= \{ g \mid I(H) = \varnothing \}$

The positive extension of the negated sentence is now computed directly as the negative extension of the contained sentence:

$$[\![\neg\,(\varepsilon^1 \wedge H\ 1)]\!]^g_+ = [\![\varepsilon^1 \wedge H\ 1]\!]^g_- = \{\, g \mid I(H) = \varnothing \,\}$$

Crucially, the output, if non-empty, is the input. This will fail to satisfy the presupposition of a subsequent sentence with a pronoun.

(14) a. It's not true that nobody is here.

b. $\neg\,(\neg\,(\varepsilon^1 \wedge H\ 1))$

Based on the equivalences we've already established, we know that the positive extension of the doubly-negated sentence *is* the positive extension of the contained positive sentence.

Doubly-negated sentences therefore introduce DRS.

$$[\![\neg\,(\neg\,(\varepsilon^1\ \wedge H\ 1))]\!]^g_+ = [\![\varepsilon^1 \wedge H\ 1]\!]^g_+ = \{\, g^{[1 \mapsto x]} \mid x \in I(H)\,\}$$

Bathroom sentences

Strong Kleene disjunction

To tackle bathroom sentences, we first need to give a semantics for disjunction; in P-DAS we do so by taking the lifted strong Kleene connective, and applying the positive closure operator.

$\vee^s$	1	0	#
1	1	1	1
0	1	0	#
#	1	#	#

Table 3: Disjunction in strong Kleene

- $\phi \vee^s \psi$ is true if either ϕ is true or ψ is true.
- $\phi \vee^s \psi$ is true only if both ϕ and ψ are false.

Lifted $\wedge^s$

$$[\![\phi \veebar \psi]\!]^g = \{\, (t \vee^s\ u, i) \mid \exists h[(t,h) \in [\![\phi]\!]^g \wedge (u,i) \in [\![\psi]\!]^h]\,\}$$

(15) a. $$[\![\phi \veebar \psi]\!]^g_+ = \{\, i \mid \exists h[h \in [\![\phi]\!]^g_+ \wedge (i,*) \in [\![\psi]\!]^h]\,\} \cup \{\, i \mid \exists h[(*,h) \in [\![\phi]\!]^g \wedge i \in [\![\psi]\!]^h_+]\,\}$$

b. $$[\![\phi \vee \psi]\!]^g_- = \{\, i \mid \exists h[h \in [\![\phi]\!]^g_- \wedge i \in [\![\psi]\!]^h_-]\,\}$$

$$\phi \vee \psi \Leftrightarrow \dagger\,(\phi \veebar \psi)$$

(16) a. $[\![\phi \vee \psi]\!]^g_+ = [\![\phi \veebar \psi]\!]^g_+$

b. $[\![\phi \vee \psi]\!]^g_- = \{\, g \mid [\![\phi \veebar \psi]\!]^g_+ = \varnothing \wedge [\![\phi \veebar \psi]\!]^g_- \neq \varnothing \,\}$

Important: one of the verification conditions for lifted $\vee^s$ involves passing the negative extension of the first disjunct into the positive extension of the second.

Since DNE is valid, we account for bathroom sentences automatically. Let's see how.

(17) a. Either there is no^1 bathroom, or it$_1$'s upstairs.

b. $(\neg\,(\varepsilon^1 \wedge B\ 1)) \vee U\ 1$

The (+/−)-extensions of each of the disjuncts:

(18) a. $[\![\neg\,(\varepsilon^1 \wedge B\ 1)]\!]^g_+ = \{\, g \mid I(B) = \emptyset \,\}$

b. $[\![\neg\,(\varepsilon^1 \wedge B\ 1)]\!]^g_- = \{\, g^{[1\mapsto x]} \mid x \in I(B) \,\}$

c. $[\![U\ 1]\!]^g_+ = \{\, g \mid 1 \in \text{dom}\ g \wedge g_1 \in I(U) \,\}$

d. $[\![U\ 1]\!]^g_- = \{\, g \mid 1 \in \text{dom}\ g \wedge g_1 \notin I(U) \,\}$

Now to compute the positive extension of the disjunctive sentence, we take the union of the positive extension of the first disjunct, and the result of passing the negative extension of the first disjunct into the second.

(19) $$[\![\neg\,(\varepsilon^1 \wedge B\ 1) \vee U\ 1]\!]^g_+ = \{\, g \mid I(B) = \varnothing \,\} \cup \{\, g^{[1 \mapsto x]} \mid x \in I(B) \wedge x \in I(U) \,\}$$

We thereby successfully account for anaphoric licensing in bathroom sentences! The sentence is predicted to be true iff there is no bathroom, or there is a bathroom and it's upstairs.

An apparent problem with this semantics is that we predict a disjunctive sentence to be externally dynamic, which contradicts the standard assumption in DS. We'll turn to this problem next.

Ignorance, disjunction, and accessibility

Groenendijk & Stokhof’s (1991) claim: disjunction is *externally static*.

(20) Either a^1 critic is in the restaurant, or we had no press.
I hope $they_1$ enjoyed it.

An apparent problem for P-DAS; disjunction should be externally dynamic (if true in the right way).

As acknowledged by G&S, *Stone disjunctions* (Stone 1992) are a problem for external staticity.

(21) Either a^1 linguist is here, or a^1 philosopher is.
(Either way) I hope she_1 enjoyed the talk.

As we'll see, P-DAS accounts for this straightforwardly.

G&S conjecture that natural language *or* is ambiguous — it can also express *program disjunction*. This is conceptually an undesirable move.

See also van den Berg (1996: ch. 2) for an argument that program disjunction is not a reasonable operation.

Problem for G&S: Rothschild disjunctions

Rothschild (2017) remarks that disjunctions cease to be externally static if the indefinite-containing disjunct is contextually entailed.

To illustrate the point, we must consider a multi-speaker discourse.

(22) a. Either a^1 critic is in the restaurant, or we had no press.

b. We had lots of press!
So, I hope they_1 enjoy their meal.

This can't be accounted for by G&S.

P-DAS predicts necessary condition on pronominal licensing: the existence of a *witness* is contextually entailed (Mandelkern 2020).

Ordinarily, disjunctive sentences (with the exception of Stone disjunctions), fail to entail the existence of a witness, due to an obligatory ignorance inference (see Simons 1996 for a related suggestion).

Locating the explanation in *pragmatics* straightforwardly captures Rothschild disjunctions; certainty may be achieved over the course of a discourse.

We assume a Stalnaker-Heim notion of *information state*, as a set of world-assignment pairs.

Information state (def.)

An *information state* c is a set of world-assignment pairs. Where, given W (the logical space):

- $c_\top := W \times \{ g_\top \}$.
- $c_\varnothing := \varnothing$

P-DAS is intensionalized in the obvious way — we add a world parameter to the interpretation function; sentences output truth-value/world/assignment *triples*.

(23) a. $[\![P\ 1]\!]^{w,g} = \{\, (\delta\ (n \in \text{dom } g) \wedge g_n \in I_w(P), w, g)\,\}$

b. $[\![P\ 1]\!]^{w,g}_{+} = \{\, (w, g) \mid \delta\ (n \in \text{dom } g) \wedge g_n \in I_w(P)\,\}$

c. $[\![P\ 1]\!]^{w,g}_{-} = \{\, (w, g) \mid \delta\ (n \in \text{dom } g) \wedge g_n \notin I_w(P)\,\}$

Update

Update of c by ϕ computes the positive extension of ϕ relative to each point in c, and gathers up the results.

Update is subject to *Stalnaker's bridge* — ϕ must be true/false at each point in c, or update fails.

Update (def.)

$$c[\phi] := \begin{cases} \bigcup_{(w,g)\in c} [\![\phi]\!]_+^{w,g} & \forall (w,g) \in c \begin{bmatrix} [\![\phi]\!]_+^{w,g} \neq \varnothing \\ \vee\, [\![\phi]\!]_-^{w,g} \neq \varnothing \end{bmatrix} \\ \varnothing & \text{otherwise} \end{cases}$$

Observation: an utterance of “P or Q” is only felicitous if P and Q are both open possibilities (Sauerland 2004, Meyer 2013).

(24) Context: *it’s common ground that someone was in the audience.*
Either someone was in the audience,
or the event was a disaster.

External staticity via ignorance ii

We can use this fact to account for the apparent external staticity of disjunction. Consider the following space of logical possibilities:

- w_{ad}: a was in the audience, and the event was a disaster.
- $w_{a\neg d}$: a was in the audience, and the event wasn't a disaster.
- $w_{\varnothing d}$: nobody was in the audience, and the event was a disaster.
- $w_{\varnothing\neg d}$: nobody was in the audience, and the event wasn't a disaster.

(25) a. Either someone^{1} was in the audience, or the event was a disaster.

b. $(\varepsilon^{1} \wedge A\ 1) \vee D\ e$

(26) $$\left\{\begin{array}{l}(w_{ad}, g_{\top}),\\(w_{a\neg d}, g_{\top}),\\(w_{\varnothing d}, g_{\top}),\\(w_{\varnothing\neg d}, g_{\top}),\end{array}\right\}[(\varepsilon^1 \wedge A\ 1) \vee D\ e] = \left\{\begin{array}{l}(w_{ad}, [1 \mapsto a]),\\(w_{a\neg d}, [1 \mapsto a],\\(w_{\varnothing d}, g_{\top}),\end{array}\right\}$$

Note, crucially, that the resulting information state is one in which 1 is *not familiar*! This means that the presupposition of a subsequent sentence with a matching free variable won't be satisfied.

External staticity via ignorance ∨

Stone disjunctions are not particularly problematic for DAS.

(27) a. Either a[1] linguist is here, or a[1] philosopher is.
$(\varepsilon^1 \wedge L\ 1 \wedge H\ 1) \vee (\varepsilon^1 \wedge P\ 1 \wedge H\ 1)$

$$(28)\quad \left\{\begin{matrix}(w_{lp}, g_{\top}),\\ (w_{l}, g_{\top}),\\ (w_{p}, g_{\top}),\\ (w_{\varnothing}, g_{\top}),\end{matrix}\right\}[(\varepsilon^1 \wedge L\ 1 \wedge H\ 1) \vee (\varepsilon^1 \wedge P\ 1 \wedge H\ 1)] =$$

$$\left\{\begin{matrix}(w_{lp}, [1 \mapsto l]), (w_{lp}, [1 \mapsto p])\\ (w_{l}, [1 \mapsto l]),\\ (w_{p}, [1 \mapsto p])\end{matrix}\right\}$$

The resulting information state is one in which 1 is *familiar*.

G&S also observe that disjunctions are *internally static*; referential information can't be passed from one disjunct to the other.

(29) # Either someone^1 is in the audience, or they're sitting down.

In DPL, the semantics of disjunction is tailored to derive this.

In P-DAS, we rule this out, again, via the pragmatics of disjunction.

Disjunctions are typically odd if the disjuncts aren't logically independent (an example: Hurford disjunctions).

(30) # Tim lives in Tokyo, or he lives Japan.

(31) $(\varepsilon^1 \wedge A\ 1) \vee (S\ 1)$

The only condition under which the second disjunct could be true, is if the first disjunct is also true; if the first disjunct is false, no DR is introduced and the second disjunct is maybe.

This means that every context in which the second disjunct is true, will be one in which the first is also true.

Logical independence iii

In order to cash out logical independence in a dynamic setting, we assume that disjunctions are subject to the following constraint:

$$\ulcorner \phi \vee \psi \urcorner \text{ is odd relative to } g \text{ if } \begin{aligned}[t] & [\![\neg\, \phi \wedge \psi]\!]^g_+ = \varnothing \\ \vee\ & [\![\phi \wedge \neg\, \psi]\!]^g_+ = \varnothing \end{aligned}$$

(29) is independently ruled out by logical independence:

$$\left[\!\left[\neg\, (\varepsilon^1 \wedge A\ 1) \wedge S\ 1\right]\!\right]^g_+ = \varnothing$$

Problems and prospects

Summary i

We've developed a new dynamic logic — P-DAS — that is *explanatory* in a way that alternatives, such as DPL, aren't.

Unlike in competing theories, the logical connectives in P-DAS are derived systematically using the following ingredients:

- A strong Kleene logical substrate.
- Implicitly, the `State.Set` monad, for passing referential information (Charlow 2019).
- A positive closure operator †, to limit DR-introduction in the negative information conveyed.

The result is a theory with *fewer stipulations* than orthox dynamic frameworks, and *superior empirical coverage*.

As a case study, we've looked at *double-negation* and *bathroom sentences*.

We also showed, once supplemented with an independently motivated pragmatic component, P-DAS is sufficiently constrained.

There's still a lot to be done...

P-DAS doesn't validate *Egli's corrolary*, but rather something weaker:

$$(\varepsilon^n \wedge \phi) \rightarrow \psi \Leftrightarrow (\varepsilon^n \wedge \phi \wedge \psi) \vee \neg (\varepsilon^n \wedge \phi)$$

This means that Donkey sentences are systematically predicted to have weak readings; this is a good prediction for certain environments (Kanazawa 1994, Champollion, Bumford & Henderson 2019).

In remains to be seen how to account for the more prevalent strong readings in this framework.

There are few other approaches to the dynamics of singular indefinites that do as much with as little, as P-DAS.

Two notable recent proposals are Rothschild 2017 and Mandelkern 2020, who develop a static semantics for anaphora. Both proposals involve certain stipulations which aren't necessary in P-DAS:

- Rothschild must assume that classically transparent conjuncts can be freely inserted.
- Mandelkern assumes that indefinites are associated with a special presupposition that is automatically accommodated.

Outlook

P-DAS arguably meets the explanatory challenge for DS as a theory of anaphora; this makes it a promising baseline dynamic logic going forward.

I'm optimistic that P-DAS can help simplify and improve accounts of other phenomena analyzed using DS, such as modal subordination, discourse plurals, donkey anaphora, etc.

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